Exam. Code : 211004
Subject Code : 4643

M.Sc. (Mathematics) 4th Semester
STATISTICS—II
Paper—MATH-587

Time Allowed—2 Hours] [Maximum Marks—70

Note :— Attempt any **four** questions. All questions carry equal marks.

1. (a) Define t-distribution and find its mean and variance.

(b) Let $X_1, \ldots, X_n$ be a random sample from normal distribution $N(\mu, \sigma^2)$. Find the distribution of

$$S^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}, \text{ where } \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

(c) If F_{n_1, n_2} represents an F-variate with n_1 and n_2 d.f. prove that F_{n_2, n_1} is distributed as $1/F_{n_1, n_2}$ variate.

2. Let $X_1 \ldots X_n$ be random sample from a continuous population with pdf.

$$f_x(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

(i) Show that $X_{(r)}$ and $X_{(s)} - X_{(r)}$ are independent, for any $s > r$.

(ii) Find the pdf of $X_{(r+1)} - X_{(r)}$

(iii) Let $Z_1 = nX_{(1)}$, $Z_2 = (n-1)(X_{(2)} - X_{(1)})$, $Z_3 = (n-2)(X_{(3)} - X_{(2)})$,........, $Z_n = (X_{(n)} - X_{(n-1)})$. Show that $Z_1, Z_2, ..., Z_n$ are independent.

3. (a) Suppose X_1 has density $f_\theta(x_1) = \frac{1}{\theta}e^{-\frac{x_1}{\theta}}, \theta > 0$ and X_2 has density $f_\theta(x_2) = \frac{2}{\theta}e^{-\frac{2x_2}{\theta}}$ and X_1 and X_2 are independent. Find sufficient statistics for θ. Find a function of the sufficient statistics which is an unbiased estimator for θ.

(b) Let $X_1, ... X_n$ be a random sample from exponential distribution with pdf.

$$f(X \mid \mu, \beta) = \begin{cases} \frac{1}{\beta}e^{-\frac{x-\mu}{\beta}}, & x > \mu \\ 0, & \text{otherwise} \end{cases}$$

Find MME and MLE of μ and β.

(c) State and prove Neyman's factorisation theorem of sufficient statistics.

4. (a) Define Uniform Most Powerful (UMP) test. Let $X_1 ...... X_n$, be a random sample from Exp(θ). Prove that UMP test for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ does not exist.

(b) Let X be an observation from population with pdf

$$f(x|\theta) = \begin{cases} \frac{1}{\theta}, & 0 < x \le \theta \\ 0, & \text{elsewhere} \end{cases}$$

For testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$, consider two critical regions $R_1 = \{x : x \ge 0.5\}$ and $R_2 = \{x : 1 \le x \le 1.5\}$. For both critical regions, find Type I and Type II error.

5. (a) Let $X_1, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$. where μ and σ^2 are unknown. Find the likelihood ratio test for testing $H_0 : \sigma^2 = \sigma_0^2$ versus $H_a : H_0 : \sigma^2 \neq \sigma_0^2$.

(b) Let $X_1, ..., X_n$ be random sample from a population with unknown mean μ and unknown variance σ^2. It is desired to test the hypothesis $H_0 : \mu = \mu_0$. Explain clearly, the procedures that you would follow to carry out the above test for one-sided and two sided problems, separately, at a given level of significance α by assuming n is large.

6. (a) Describe applications of a chi-square distribution for testing the goodness of fit and for testing the independence by clearly stating the problems, the assumptions and hypotheses involved in these.

(b) Write note on paired t test.

7. (a) State and prove Gauss Markoff's Theorem.

(b) Define estimable linear parametric function. For the general linear model of full rank $Y = X\beta + \in$; $E(\in) = 0$ and $E(\in\in') = \sigma^2 I$, prove that every linear parametric function is estimable.

8. (a) Give complete analysis of one way classified data for fixed effect model.

(b) Let Y_1, Y_2, Y_3, Y_4 be random variable with $E(Y_1) = E(Y_2) = \theta_1 + \theta_2$, $E(Y_3) = E(Y_4) = \theta_1 + \theta_3$ determine the estimability of the following linear parametric functions :

(i) $2\theta_1 + \theta_2 + \theta_3$ (ii) $\theta_3 - \theta_2$.